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Packet Radio Note No.

Packet Radio Note: Packet Radio, Spread Spectrum, Delay, and Throughput

I INTRODUCTION

As the computer invades all corners of civilian and military life, remote users of computer resources proliferate. The demand for networks to connect fixed users to computers has been recognized, commented on, and even over-reacted to in some circles. Recently a demand to connect portable and mobile digital terminals into a network with a centralized computing resource has been felt, and efforts to satisfy this demand have been initiated.

Perhaps the only reasonable connection to mobile users is a radio link. If we applied radio to meet the demand, at first we might simply assign a mobile radio channel to each user; however, as the number of users grew we would quickly run out of spectrum. Since most digital terminals have very low duty cycles we would be using the spectrum very inefficiently if we dedicated a high data-rate channel to each user; yet we could not use low data-rate channels commensurate with the average user rate, because this would result in entirely too much delay, and the advantage of a mobile computer resource connection would be lost. Some way to share a high data-rate channel amongst many users with "bursty" use statistics is needed. The packet radio concept as implemented in the University of Hawaii ALOHA network was devised to provide such a channel. This concept must compete with others for economic viability, and it must satisfy user needs to be successful. Two important measures of performance of radio networks to connect digital terminals must be the average message delay, and the number of users per radio channel.

In this note we compare two related measures for several alternative packet radio signalling schemes. We determine the average message delay, normalized by the message duration and the average throughput per unit bandwidth. In all schemes, the message delay can be reduced by reducing

throughput, thus the two variables are tradeoff variables and are presented as a set of curves. From these curves the average number of users which any radio channel will support can be determined using the average data rate per user and the channel bandwidth. Similarly the average message delay can be determined from channel bandwidth, message length, and the normalized message delay.

We begin by analyzing a single-channel ALOHA-type signalling scheme using spread-spectrum waveforms. We find that the delay-throughput equations describing the performance of this scheme are very much like the equations for a narrow-band (conventional modulation, such as PSK) ALOHA channel; however, the bandwidth for a given data-rate throughput and delay is much larger. Thus the single-channel ALOHA spread-spectrum scheme is not an appealing choice. When we consider the effects of typical multipath propagation channels, the situation is even less favorable for single channel spread spectrum.

Next we consider the modification of this scheme by adding a sync preamble, synchronizing packets (slotting the channel), and sequentially varying the spread-spectrum waveforms. Although each of these techniques improves efficiency, none provides the efficiency of conventional ALOHA.

Finally we consider multi-channel spread-spectrum schemes in order to improve throughput efficiency. Since we know of no way to assure orthogonal channels without synchronizing all terminals so accurately that mobile units would be impractical, we have analyzed non-orthogonal channels, where the channel power-rejection ratio is inversely proportional to the spreading-factor of the spread spectrum signal. We find that even such multiple-channel systems are much less efficient than conventional ALOHA channels.

Since there are many other tradeoff dimensions which we have not analyzed we do not draw any conclusions or recommend any particular access mode. The curves of delay versus bandwidth utilization (normalized throughput) are presented as one component in the overall tradeoff study which is yet to be completed.

II SOME SIMPLE SPREAD SPECTRUM SYSTEMS

A. User Model

Throughout this note we use the model suggested by Abramson.[1]* Briefly we assume an infinite population of identical, unperturbable users contending for a single r.f. channel. All traffic is sent one-way from the user to a central receiver which would be a repeater in a packet radio network. We ignore the effect of external interference caused by other nearby repeater-terminal mini-networks and we ignore return traffic and network protocol signals such as acknowledgments. We assume the users generate messages and retransmissions with exponentially-distributed intergeneration times and an aggregate traffic rate λ messages/second.

B. Channel Model

Initially we consider an ideal r.f. channel. We ignore the effects of noise and propagation, and assume that all packets generated are received with equal strength at the central receiver.

Later we consider the effects of multipath, noise, and propagation attenuation. The specialized models used are described as needed.

C. Signal Model

To provide a focus for analysis we have selected a form of spread-spectrum transmission using binary-orthogonal signalling and non-coherent reception. Since we initially ignore noise and multipath, the results we obtain will be independent of the form of modulator/receiver and depend only on the processing gain, or time-bandwidth product; hence they will be generally applicable to a wide variety of spread-spectrum signals.

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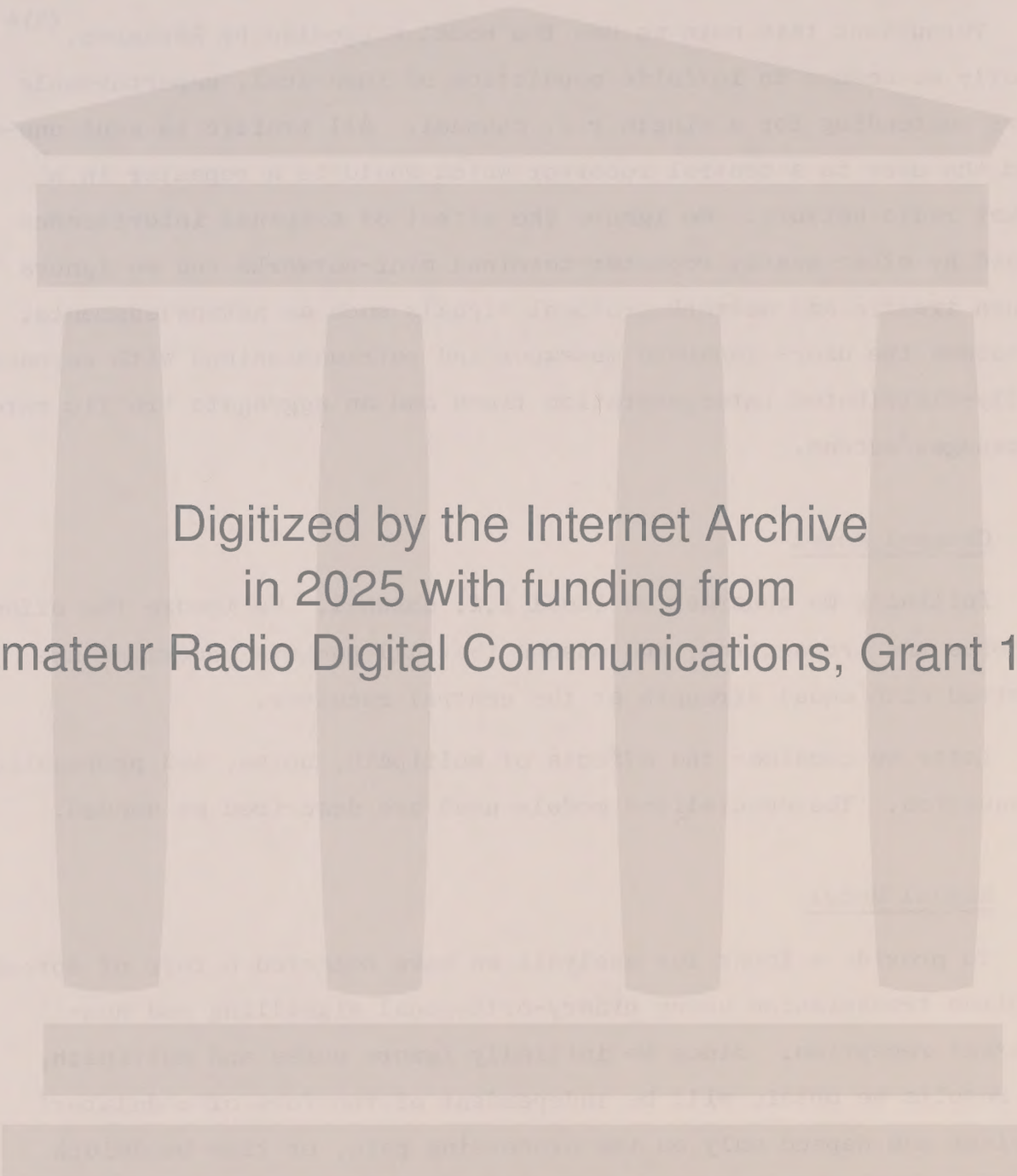
* References are listed at the end of the note.



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https://archive.org/details/spread-spec-delay-through

The spread-spectrum system which we will consider operates as shown in Figure 1. An encoded binary data-stream is delivered to the modulator whenever a message (packet) is sent. The modulator selects one of two wideband signals depending on whether the message bit is a "one" or a "zero." The wideband signals are "spread-spectrum" in the sense that the bandwidth of each signal is much greater than the minimum bandwidth required to send the information at the basic bit rate. Let the signal r.f. bandwidth be W and the bit duration be T. Then the signal is spread-spectrum in the sense that $TW \gg 1$. The parameter $N = 2\ TW$ is a measure of the degree of "spread" and $\sqrt{N}$ is commonly called the processing gain of the signal. A bound on the number of orthogonal waveforms of duration T which exist in a band W is $N + 1$.

The receiver consists of a pair of matched filters--one matched to each signal. The filter outputs are envelope-detected and subtracted. The result is sampled synchronously with the received bit-train and a decision is based on the sample.

Suppose that a single bit, say a "one" is transmitted. Then the envelope of the output of the filter matched to S_1 will have a peak "lobe" and many "side-lobes" something like the sketch in Figure 2. The envelope of the filter matched to S_0 will have many little "side-lobes" but no peak. In fact, if S_1 and S_0 are truly orthogonal, the envelope of the S_0 output will be zero (except for noise) at the moment when the S_1 output is maximum. Decisions are made only at moments when one or the other filter is a maximum (this is the sampling instant) hence decisions will be made correctly unless noise, or some interfering signal causes the envelope of the wrong filter to exceed the envelope of the correct filter.

If an interfering signal occurs simultaneously with the desired signal, it will cause no errors unless it occurs approximately in synchronism with the desired signal. The situation might look something like the sketch of Figure 3. Well designed spread-spectrum signals will have a major lobe with width roughly $\frac{2}{W} = \frac{4T}{N}$. Whenever the two signals have major lobes which are offset by less than $\frac{4T}{N}$ second there will be

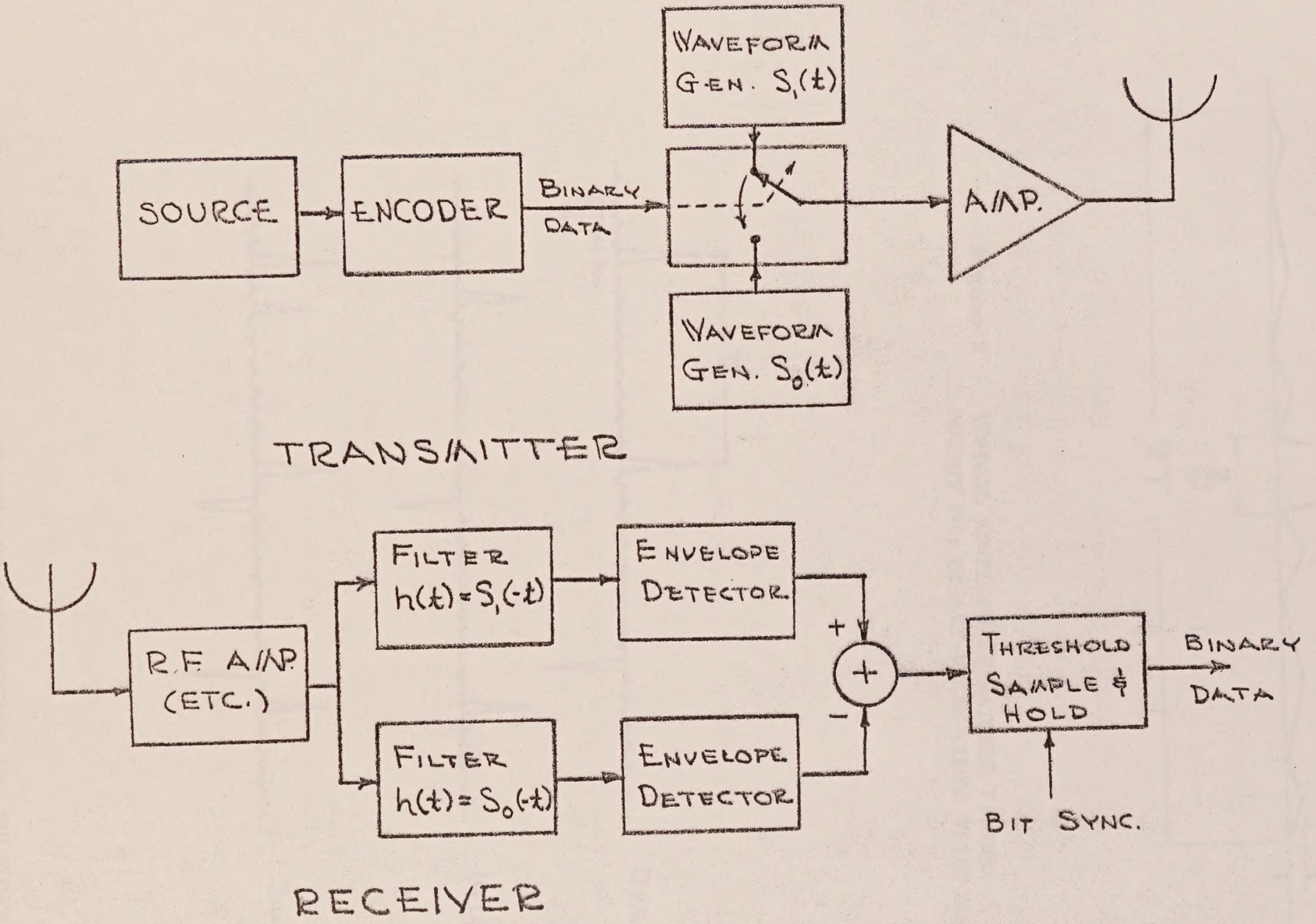


Figure 1 SPREAD-SPECTRUM MODEL

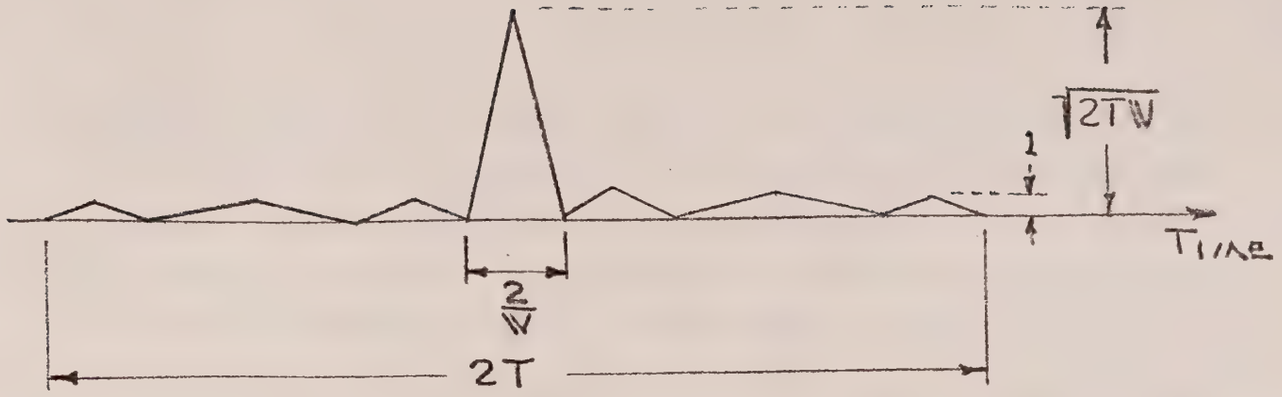


Figure 2 TYPICAL ENVELOPE OF MATCHED FILTER OUTPUT FOR BINARY-PHASE-CODED WAVEFORM

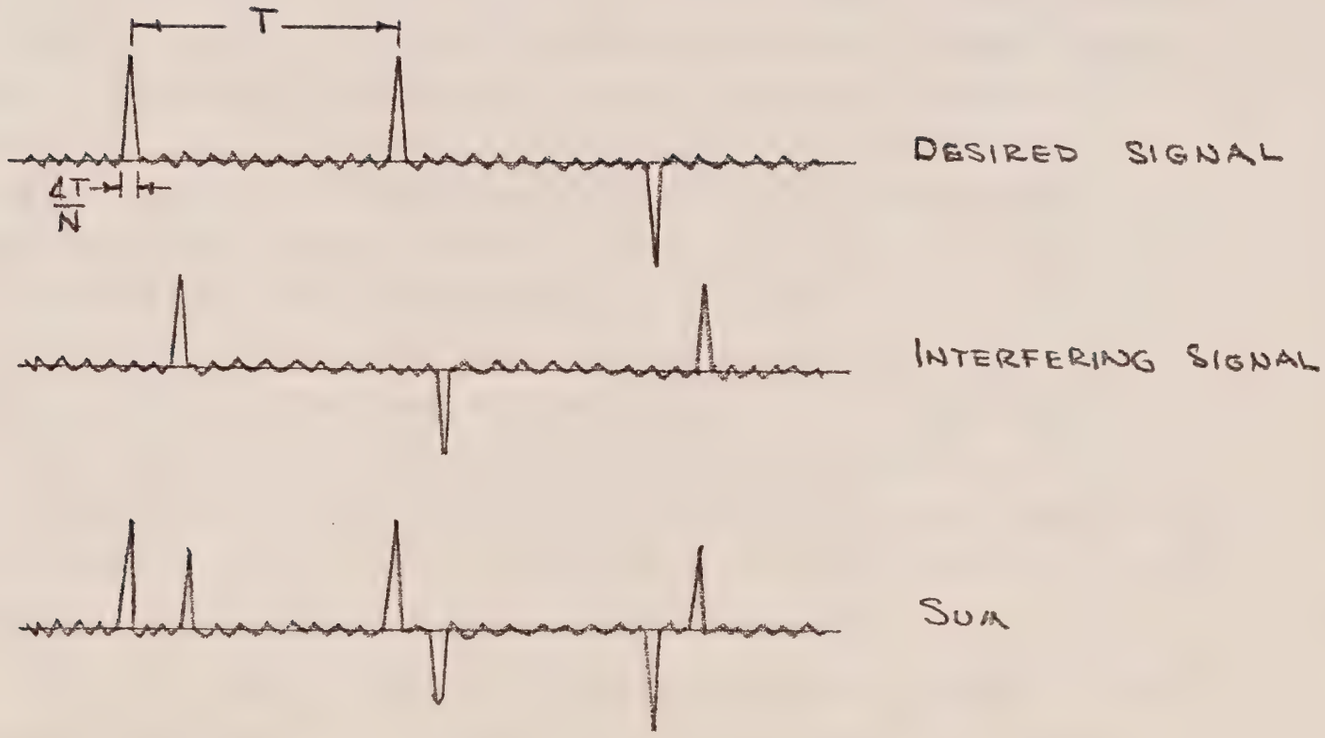


Figure 3 EFFECT OF INTERFERENCE (NO MULTIPATH)

increased probability of error. Thus, we can define the event "interference" in terms of the occurrence of two signals with bit-start times within some fraction of $\frac{4T}{N}$ second of each other. Let this fraction by $\frac{k}{2}\frac{T}{N}$, where k is a small constant, on the order of 4, which depends on the details of waveform design, filter design, modulation format, etc.

Notice that the same definition of interference holds for any fixed-waveform, single-channel spread-spectrum system using binary signals, whether they are orthogonal or antipodal and independent of the detection/decision mechanism. If M-ary signals were used similar results would hold, but it would be necessary to interpret T as the symbol time. Varying, or multiple-waveforms and multiple-channels will be considered in Section III.

D. Performance in the ALOHA Mode

There are many ways in which we can use this basic spread-spectrum technique. First, we consider the ALOHA random-access scheme. Recall that in this scheme a user simply transmits his packet whenever it is generated. If no other user transmits a packet so as to interfere, the packet is successfully communicated and a positive acknowledgment is generated by the central receiver. If the packet is interfered with by another packet, no acknowledgment is given and the user terminal retransmits the packet at some later time chosen randomly. Thus, more packets are transmitted then successfully received. Abramson has analyzed this scheme in Reference (1), and various modifications of this analysis may be found in the packet radio note series. The static performance of such an access scheme can be determined from the relationship of throughput to total traffic, rederived below.

When this mode is used with narrowband signalling schemes we must consider the channel to be occupied completely whenever a packet is being transmitted. Thus, to be successful, a user must be lucky enough to start transmission more than one message duration after the last prior packet start and his luck must hold so that no other user starts while he is transmitting. The probability that this will occur is just

the probability that the user will choose a period of duration two packet lengths long which contains no other packet start times. Call this the probability of success, P_s, and let the message (packet) last T_m seconds and suppose that the average aggregate transmission rate is λ packets per second. Then since the process is assumed to be generated with exponential interarrival times the probability of zero packets in $2\ T_m$ seconds is just

$$P_s = e^{-2\lambda T_m} \quad .$$

If we consider only the time dimension of the channel, then ideal utilization would allow transmission of $\frac{1}{T_m}$ packets/second. The ALOHA channel allows λP_s packets/second. Hence the ratio of ALOHA utilization to ideal (time) utilization is $\lambda T_m P_s$. This ratio is called the throughput and designated S. The product λT_m is called the traffic, G, and we have for a narrow-band ALOHA channel

$$S = Ge^{-2G} \quad .$$

When spread-spectrum signals are used, it is possible that two signals may occur simultaneously without interference as described above, provided that the two users were lucky enough to choose start-times which did not cause overlap of the main-lobes of the filter response to each signal. Thus, the probability that a later packet will interfere with an early packet is reduced greatly for large N; however, notice that the later packet cannot be received by a receiver operating on an early packet without loss of the early packet. The receiver can be busy when a packet starts. Thus, there are two events which can cause an unsuccessful transmission. We define these as:

$$B \equiv \text{the receiver may be busy, and}$$

$$I \equiv \text{the signal may be interfered with.}$$

The first event will occur if one or more signals have started within the T_m seconds prior to packet start. The second event will occur if

one or more packets start in any of the T_m/T "windows" of width $\frac{kT}{N}$ seconds following packet start.

The probability of success is the probability that neither of these events occur, and it can be derived as below.

$$P_s = 1 - P(B \cup I) = 1 - P(B) - P(I) + P(B \cap I) \quad .$$

Since the events B and I are independent (due to the Poisson point process generating the start times),

$$P_s = 1 - P(B) - P(I) + P(B)P(I) \quad .$$

From the Poisson assumption

$$P(B) = 1 - e^{-\lambda T_m} \quad ,$$

and

$$P(I) = 1 - e^{-\lambda \left(\frac{kT}{N}\right)\left(\frac{T_m}{T}\right)}$$

so that

$$P_s = e^{-\lambda T_m \left(1 + \frac{k}{N}\right)}$$

The channel throughput, S, is just

$$S = Ge^{-G\left(1 + \frac{k}{N}\right)}$$

Notice that if $N \rightarrow k$ (narrowband signal) we have the equation for a classic ALOHA channel, and if $N \rightarrow \infty$ (very wideband signal) we have the equation for a simple slotted ALOHA channel. For $k \leq N \leq \infty$ the channel throughput will lie between ALOHA and slotted ALOHA.

This result is mildly surprising. One might expect that as $N \rightarrow \infty$, perfect time-capture would occur so that the throughput should approach 1 asymptotically. This would indeed be the case if we allowed a separate

receiver for each user; however, with only one receiver, the throughput cannot exceed that of slotted ALOHA. This is because the channel can be "busy" with either a valid message, or with interference. For example, suppose that the receiver is handling a valid message, and a second terminal generates a message. Then if $N \rightarrow \infty$ no interference will occur, but when the valid message ceases, the interfering message will capture the receiver and hold it until the interfering message is complete. The receiver will be "busy" handling traffic which does not contribute to throughput, hence throughput will not approach one.

In later sections of this note we will consider ways to increase the throughput by reducing the busy probability. This can be done in several ways, each of which complicates the system and the analysis considerably.

E. The Effect of Multipath

We have shown that the throughput for a spread-spectrum system using an ALOHA access mode can be determined from the equation

$$S = Ge^{-G\left(1 + \frac{k}{N}\right)}$$

This equation depends on the assumption that two signals will interfere whenever they are "synchronized" at the receiver so that individual bits arrive within a time window of width T_w where

$$\frac{kT}{2N} = T_w$$

Thus the equation could have been written

$$S = Ge^{-G\left(1 + \frac{2T_w}{T}\right)} .$$

This is valid only so long as $T_w \leq \frac{T}{2}$. When $T_w = \frac{T}{2}$, there is no possibility of "interlace" of two signals and spread-spectrum will provide no improvement over narrowband signals.

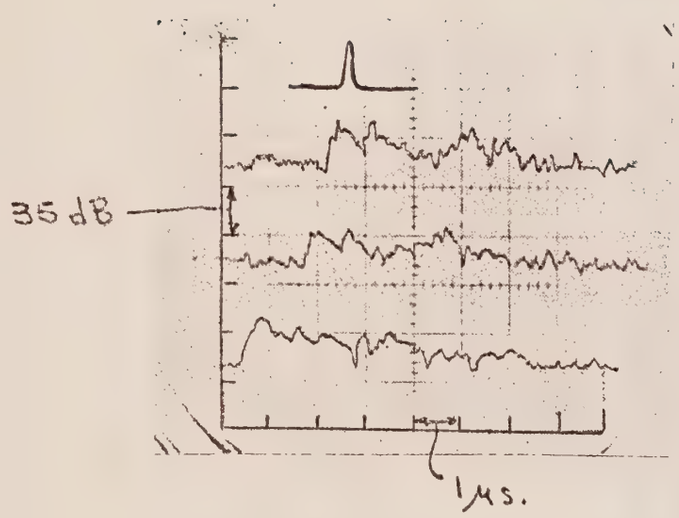
In the frequency bands being considered (center-frequency > 400 MHz) the effect of multipath on any type of signal is to stretch the duration of the signal by echoing it.

For example, if we transmit a very short (300 ns) r.f. pulse from the hall of Justice in San Francisco we will receive a much longer pulse in the financial district [Professor G. Turin[2] performed this experiment with the results shown in Figure 4.] The received pulse appears to be the summation of many of the transmitted pulses, each delayed by a random interval, and each with a random gain and phase. This channel appears to be well modeled by a delay line with random gain taps set at random points.

This model is physically appealing, since we can easily visualize a pulse of energy radiated in all directions from the transmitter, and bounding off various surfaces like a beam of light from a mirror. Part of the energy may find a direct path to the financial district, and there are enough reflecting surfaces so that some of them will be oriented to reflect additional copies of the pulse to the district. Notice that Turin's measurements do not show isolated reflections, but indicate a more continuous variation. This is in part due to the fact that the pulse duration is long relative to typical off-path delays; however, it is doubtful that any pulse of reasonable duration (or any other waveform of bandwidth less than a few GHz) would resolve the individual paths. Other measurements made by Turin[2] and Cox[3] support this model, and SRI will be making additional measurements to determine specific parameters of the channel of importance to the packet radio network.

Thus, we can see that the effect of multipath on spread-spectrum system performance is to increase the size of the time-window for interference. This time-window will correspond roughly to the 3-dB duration of the impulse-response of the multipath channel. We will call this the delay-spread and designate it T_s. Then for reasonably wide-band spread-spectrum signals with bit duration $>2\ T_s$ we have

$$S = Ge^{-G\left(1 + \frac{2T_s}{T}\right)}.$$



Transmitted Pulse Shape

Received Pulse - 2920 MHz

Received Pulse - 1280 MHz

Received Pulse - 488 MHz

Figure 4 EXAMPLES OF MULTIPATH PROFILES IN SAN FRANCISCO--
COPIED AND RETOUCHED FROM TURIN et al., op. cit.

We can readily see that in many practical cases the throughput of ALOHA spread-spectrum will be determined by the multipath, not by the spreading factor. Furthermore, in these cases it will always be less than the throughput for a slotted ALOHA mode, and usually not be better than simple ALOHA.

F. Performance Summary--ALOHA Spread Spectrum

The relationship of throughput and traffic can be very illuminating, particularly if we wish to compare the performance of several access modes. These relationships can be used to derive the average packet delay time, by noting that S valid packets generate G transmitted packets, so that on the average one packet will take $T_m G/S$ seconds* for successful transmission. Curves of delay versus throughput are given in Figure 5 for spread-spectrum signals operating in various multipath environments. Also shown for comparison purposes are the limiting ALOHA and Slotted ALOHA curves.

It is important to notice that the throughput scales on this figure are in terms of throughput at the base-band bit rate. For spread-spectrum signals the throughput in bits/Hz can be obtained by dividing S by N. For narrow band signals using QDCPSK the throughput in bits/Hz is obtained by reading S directly.

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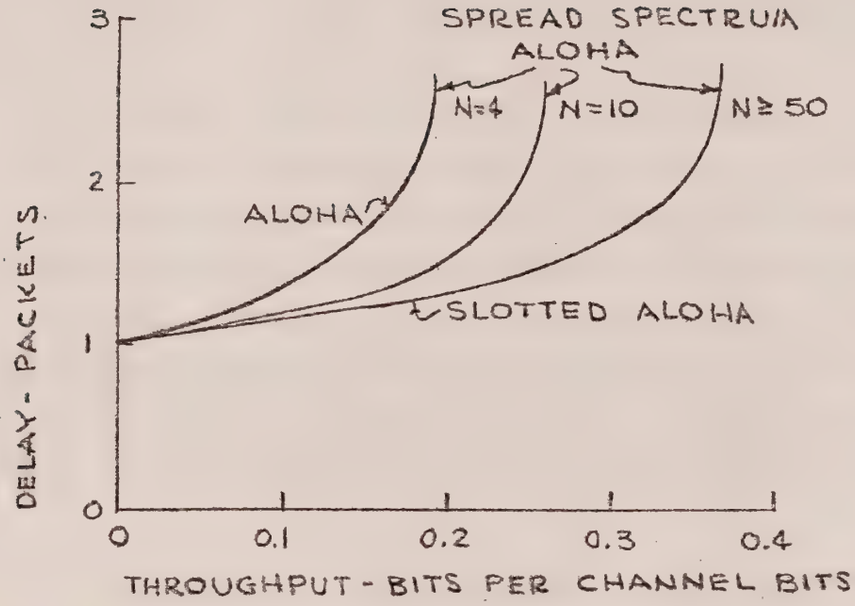
* This ignores propagation time, acknowledgment time, and time necessary for waiting but these are small and will not affect relative delay for different access modes significantly. For a detailed account of delay time in an ALOHA net see



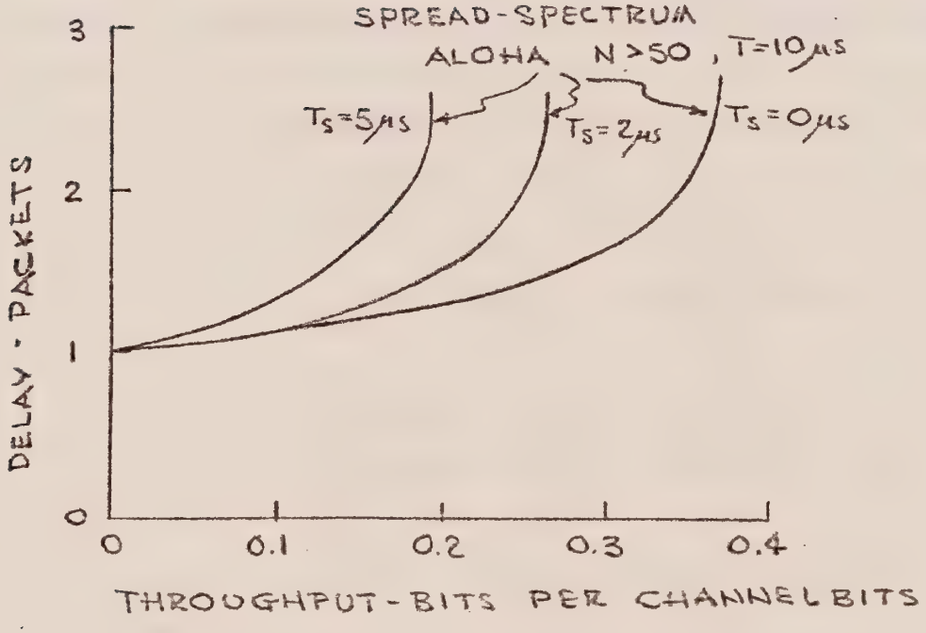


Figure 5 DELAY-THROUGHPUT TRADEOFFS FOR SIMPLE SPREAD-SPECTRUM SYSTEMS

III MORE COMPLEX SPREAD-SPECTRUM MODES

In the previous section we concentrated our attention on a simple ALOHA system using spread-spectrum signals. We found that the use of spread-spectrum can provide only a limited improvement over narrow-band ALOHA techniques particularly when realistic multipath channels are considered. This limitation comes about partly because of the multipath, and partly because the receiver can be busy with invalid traffic. In this section we describe and analyze several techniques which can overcome these limitations at the cost of equipment complexity.

A. Spread-Spectrum With a Synchronizing Preamble

We have so far assumed that the receiver could be "busy" with useless traffic. This was because we assumed that any signal could capture the bit-sync circuit, so that if two non-interfering packets overlapped, the second packet would capture the receiver sync circuit after the first packet released it. In TPRN No. 33, McGuire suggested a sync preamble to precede every packet. Such a preamble has certain associated difficulties since it must be 6 to 10 bits long, and SAWD's of this length may be impossible to produce at bit rates of 100 kBps. Nevertheless, some form of synchronization is possible and very desirable, since in this case the receiver need never be busy with useless traffic.

We can analyze this case by defining events

$C \equiv$ Sync capture

$I \equiv$ Packet or sync interfered with .

A packet will be successful if the event $C\,\bar{I}$ occurs. P(I) will be given as before, by

$$P(I) = 1 - e^{-\lambda T_m \left(\frac{k}{N}\right)}$$

(assuming that the sync interference window is no larger than the bit interference window). To determine P(C), consider the sequence of packet start-times as seen by the receiver. Let our time origin be placed at a successful sync capture. Then any packet with a start time prior to T_m seconds will not capture the sync, and the first packet with a start time after T_m will capture the sync circuit [although the capture may be interfered with as accounted for in P(I)]. The waiting time after the current packet until the next start time is an exponential random variable with mean $1/\lambda$, hence the average time between successful sync captures will be $T_m + 1/\lambda$. The average time between packet starts is $1/\lambda$, so that the probability of capture is

$$P(C) = \frac{1/\lambda}{T_m + 1/\lambda} = \frac{1}{\lambda T_m + 1} = \frac{1}{G + 1} \quad .$$

The probability of successful packet transmission is

$$P_s = P(C)\ [1 - P(I)]$$

$$= \frac{e^{-G\left(\frac{k}{N}\right)}}{1 + G}$$

and the throughput is

$$S = \frac{G}{1 + G}\, e^{-G\left(\frac{k}{N}\right)}$$

It is interesting to find the maximum throughput possible for large N. This occurs for G such that $\frac{\partial s}{\partial G} = 0$:

$$\frac{\partial s}{\partial G} = \frac{1}{(1 + G)^2} - \frac{k}{N}\,\frac{G}{1 + G}\, e^{-G\left(\frac{k}{N}\right)}$$

has roots at

$$G = \frac{-1 \pm \sqrt{1 + 4\,\frac{N}{k}}}{2} \quad .$$

For large N, S_{max} occurs at

$$G \doteq \sqrt{\frac{N}{k}} \quad ,$$

so that

$$S_{max} \doteq \frac{\sqrt{N/k}}{(1 + \sqrt{N/k}) \exp (\sqrt{N/k})}$$

for large N

$$S_{max} \doteq \left(\frac{1}{e}\right)^{\sqrt{\frac{N}{k}}} \quad .$$

Thus, the addition of a sync preamble, which is necessary to satisfactory operation anyway, will aid time-capture and allow the throughput of spread-spectrum ALOHA to approach 1 for large spread-factors if we ignore multipath.

Multipath can be accounted for, as before, by replacing k/N by $\frac{2T_s}{T}$.

If we make this substitution we find that

$$S = \frac{G}{1 + G} e^{-G\left(\frac{2T_s}{T}\right)}$$

B. Slotted Spread Spectrum

A slotted ALOHA system is one in which the terminals are all synchronized so that transmissions only occur in time slots, roughly of one packet duration. This synchronization roughly doubles the throughput relative to an unslotted system because the "time-window" for interference is halved. Only messages generated during the T_m seconds prior to a slot can be transmitted and interfere, so that the throughput for a narrowband slotted ALOHA system is

$$S = Ge^{-G}$$

Such simple analysis is possible for a narrowband slotted system because once a terminal initiates a packet it occupies the time-slot completely. A spread-spectrum signal is more difficult to analyze since the time-slot is only occupied part of the time, and only signals which are approximately synchronized will interfere. Different propagation times will assure that signals will not occur simultaneously, and for large enough values of N this fact can be used to improve performance when there is no multipath. For analysis we make two key assumptions. First, we assume that the first signal received will capture the synchronization circuits so that the receiver will be busy or blocked to all later arrivals. We will consider multiple-receiver systems later. Second, we assume that all terminals generate and retransmit traffic with the same average rate. This latter assumption is not precise since we know that the closer terminals with shorter propagation time will be blocked less often than the more distant terminals; however, the closer terminals will also perceive more interference. These two effects will be partially compensating.

Suppose that all the terminals are located in some circle of radius R with the central receiver at the center. In practice R will be determined by the terminal radiated power, the noise level, and the attenuation losses as well as by tolerable error rates. Let r be a variable range to an arbitrary terminal in this circle. Assume that a terminal captures the bit-sync circuit of the receiver if his first bit is not interfered with. Then all terminals at ranges less than $r - \frac{cTk}{2N}$ can capture the receiver before the terminal at r can propagate a packet to the receiver (assuming that everyone has a synchronized clock, which they will not have in practice). The set of blocking terminals will generate traffic at a rate $\left(r - \frac{cTk}{2N}\right)^2 \frac{\lambda}{R^2}$, so that the busy probability will be

$$P(B) = 1 - e^{-\frac{\left(r - \frac{cTk}{2N}\right)^2}{R^2} \lambda T_m}$$

Terminals at ranges such that the bit start times will overlap within $\pm \frac{T}{N} \frac{k}{2}$ of the terminal at range r will cause interference. Thus, if we draw two circles, one at range $r + \frac{cTk}{2N}$ and one with range $r - \frac{cTk}{2N}$, these circles will enclose all terminals which might interfere by starting during the first bit of a packet transmitted by a terminal at r. (The velocity of light is designated by c.) These terminals will generate traffic at a rate

$$\left[\frac{\left(r + \frac{cTk}{2N}\right)^2 - \left(r - \frac{cTk}{2N}\right)^2}{R^2}\right] \lambda$$

Similarly the terminals in an annular ring centered at r + cT of width $\frac{cTk}{2N}$ will be potential interference sources since they are also synchronized, but start during the second bit. There will be such an annular ring for each bit. Those rings which lie within the maximum range of the central receiver will all contribute to interference, and there will be n_r such rings, where

$$n_r = \left[\frac{R - r}{cT}\right]$$

and $[\cdot] \equiv$ largest integer smaller than $(\cdot)$. Thus, the probability of interference to a terminal at range r is

$$P(I,r) = 1 - e^{-\sum_{i=o}^{n_r} \frac{2(r + icTk)cT}{NR^2} \lambda T_m}$$

Considering the busy probability and the interference probability, the success probability for a terminal at r is:

$$P_s(r) = e^{-\lambda T_m f(r)} ,$$

where

$$f(r) = \frac{\left(r - \frac{cTk}{2N}\right)^2}{R^2} + \sum_{i=o}^{n_r} \frac{2(r + icTk)}{NR^2} cT \quad .$$

If the terminals are uniformly distributed over the circle of radius R, then the probability of finding one at r is

$$p(r) = \frac{2r}{R^2} \qquad 0 \le r \le R$$

$$= 0 \qquad \text{elsewhere}$$

In this case the throughput is given by integrating over the variable r. In general, the traffic rate G would be a function of r; however, we have assumed that G is constant so that the throughput is simply

$$S = G \int_0^R e^{-Gf(r)} p(r)\, dr \quad .$$

If we substitute for f(r) and p(r) in this integral we obtain an integral which depends upon n_r, hence cannot be integrated except numerically. A lower bound on throughput can be found by replacing the function n_r by its maximum value which is $\frac{R}{cT}$. The lower bound can be integrated in terms of the complementary error function; however, the result is not particularly illuminating. This is a good bound for large N. Curves of this bound have been calculated and will be presented for discussion later.

If we consider the operation of such a system in a multipath environment, we need only replace $\frac{k}{N}$ by $\frac{2T_s}{T}$. For large delay-spread multipath, such as might occur in an urban area, and typical bit-rates of 100 kBps or greater, $\frac{2T_s}{T}$ may approach 1, and the slotted spread-spectrum system performance will approach the performance of a narrow-band slotted system. For other values of $\frac{2T_s}{T}$, the bound derived above applies with a suitable interpretation.

C. Single Channel-Multiple Waveform Spread-Spectrum

So far we have dealt with systems which require only one or two spread-spectrum waveforms and therefore only one or two filters. The system complexity has been concentrated in methods to obtain a preamble

sync or a slot sync. Several alternatives exist. The first to be discussed is called single-channel multiple-waveform. In this system each terminal sends a sequence of different waveforms for each packet. The receiver must have a matched filter for each waveform. We must abandon the simplicity of a binary noncoherent receiver and assume that some form of differentially-coherent signalling is used so that we only need one waveform per bit. We restrict the receiver to the ability to handle only one packet at a time, so the receiver is single-channel. If we use M waveforms, and M is smaller than the number of bits per packet, then we repeat the waveform sequence during the packet.

In this system interference between two packets will occur only when they are synchronized within the first $\frac{kT}{N}$ seconds of the first bit of the sequence of waveforms. The busy probability can be reduced by using the first few bits of the packet as a sync preamble, so that the results we derived for the preamble-synchronized spread-spectrum system apply, except that there are $\frac{1}{M}$ as many possibilities for interference. Hence the interference probability will be reduced to

$$P(I) = 1 - e^{\frac{-Gk}{MN}} \quad .$$

The throughput will be increased to

$$S = \frac{G}{1 + G} e^{\frac{-Gk}{MN}} \quad .$$

Notice that even in this case which requires a very complex receiver, throughput cannot exceed $\left(\frac{1}{e}\right)^{\sqrt{1/MN}}$.

In this case multipath again limits throughput by broadening the interference window. However, because the code is changed at each bit, interference can only occur in one window out of M. Interfering packets starting in the last M-1 windows will tend to decrease the effective signal-to-noise ratio and may cause occasional errors, however for present purposes we will ignore this problem (but see the next section).

D. Performance Summary

Curves of delay versus throughput are given in Figure 6 for each type of system discussed in this section. Typical values have been chosen for necessary system parameters. These are shown on the curves.

It is important to notice that the throughput scales on this figure are in terms of throughput at the base-band bit rate. For spread-spectrum signals the throughput in bits/Hz can be obtained by dividing S by N. For narrow band signals using QDCPSK the throughput in bits/Hz is obtained by reading S directly.

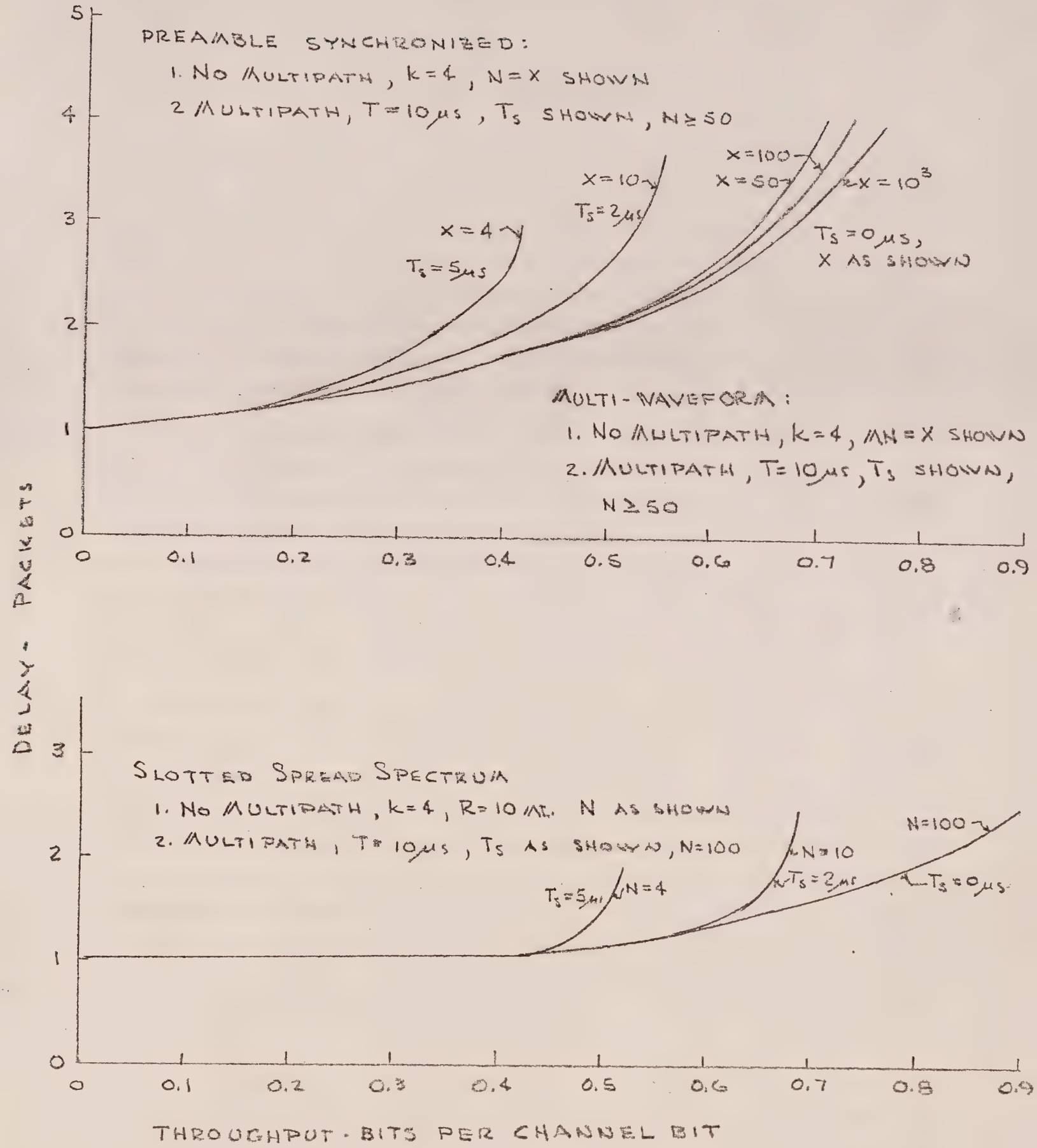


Figure 6 DELAY-THROUGHPUT TRADEOFFS FOR MORE COMPLEX SPREAD-SPECTRUM SYSTEMS

IV MULTIPLE CHANNEL SYSTEMS

We have discovered so far that spread-spectrum systems can provide throughput approaching 1 if we allow moderately complex systems (preamble-synchronized or slotted ALOHA) and very large bandwidth ($N \rightarrow \infty$). We might therefore conclude that spread-spectrum systems were superior to narrowband systems. But the proponent of narrow band systems would argue that our spread-spectrum systems require much more bandwidth and that throughput exceeding 1 can be obtained using multiple narrow-band systems occupying the same bandwidth hence narrow-band systems are superior.

Thus, the question of the efficient use of the r.f. spectrum becomes central to a comparison of spread-spectrum and narrow-band systems. It is clear that single-channel spread-spectrum systems do not use the spectrum efficiently in the sense that they occupy much more spectrum than narrow-band schemes in order to accommodate a given number of users. However, the spread-spectrum format lends itself readily to multiple channel operation; so in order to fairly compare the two approaches, we must determine just how many multiple data channels can be placed in the same r.f. channel.

At first one might be tempted to believe that the answer to this question depends upon how cleverly the spread-spectrum signals are generated, and indeed this may be true. For example, if we could somehow generate orthogonal signals for each channel, then the answer to the how many channels question would be bounded above by N - 1. One means of maintaining orthogonal channels is by frequency-division--but this is just the narrow-band approach from another point of view. If we consider spread-spectrum to imply that each <u>bit</u> is frequency spread, then we must have synchronization within all terminals of all channels to a very small fraction of a bit to be able to generate orthogonal waveforms. For a 2 TW product of N, the synchronization accuracy required would be on the order of T/N (recall that T = bit duration, W - r.f. bandwidth). Since differential propagation time between terminals and the central receiver will usually exceed T/N for even moderate N, such synchronization is out of the question for mobile terminals. It would be extremely difficult even for fixed terminals. Hence, we cannot safely assume orthogonal channels.

Lacking a clever means to obtain orthogonal channels, we are forced to assume that the channels are at least partially interfering, and analyze a model network to determine how many data channels can be placed in a single r.f. channel before the interference becomes intolerable.

This is a simple calculation, once we agree upon some reasonable ground rules. Those used for this analysis are:

1. All terminals radiate a fixed power sufficient to achieve a specified maximum bit error rate at a specified range R_o in the presence of noise and multiple channels.

2. Terminals are uniformly distributed over an annular area centered on the central receiver, with minimum radius ϵ and maximum radius R_o.

3. The power received in each channel caused by an interfering-channel signal is reduced by a factor of $\frac{1}{N}$ from the interfering-channel signal level.

Items 1 and 2 are self-explanatory. To explain item 3, suppose that we generate our spread-spectrum signals by using a random binary sequence to bi-phase modulate a carrier at a rate of $\frac{W}{2}$ bits per second, then the individual waveforms will be given by

$$S_j(t) = \sum_{i=1}^{2TW} a_{ij} \cos(wt)\, p\left(t - \frac{2i}{W}\right)$$

where

$$p(t) = 1 \qquad 0 \le t \le \frac{2}{W}$$

$$= 0 \qquad \text{elsewhere}$$

and the independent random variables a_{ij} take on values +1 or -1 with equal probability. The matched-filter response will be

$$V_{jj} = \frac{1}{T} \int_o^T S_j(t)\, S_j(t)\, dt = \frac{1}{2}$$

and the cross-channel response of filter j to unsynchronized signal k will be

$$V_{kj}(\tau) = \int_0^T S_j(t) S_k(t - \tau) dt \quad ,$$

where τ represents the lack of synchronization. It is reasonable to assume that τ has a uniform distribution over an entire bit. If we solve for $V_{kj}(\tau)$ and average over τ we have

$$\overline{V_{kj}(\tau)} = 0$$

and

$$\overline{V_{kj}^2(\tau)} = \frac{1}{4N}$$

Thus, the ratio of cross-channel power $(\overline{V_{kj}^2})$ to desired channel power (V_{jj}^2) is $\frac{1}{N}$.

There are many other ways to generate spread-spectrum signals, for example we could use 2N - 1 sequences generated by a maximal-length shift register of period greatly exceeding T as the a_{ij} sequence. We could also choose 2N - 1 noise waveforms of bandwidth W and duration T as the signal waveforms. Both of these choices would result in a ratio of cross-channel power to desired power of $\frac{1}{N}$, when we average over all possible synchronizations of the two waveforms. In fact, we know of no way to choose a set of waveforms which will provide a lower ratio of cross-channel to desired channel power in an unsynchronized system, although one may exist.

With these ground rules stated, let us focus attention on the network. Let N_c be the number of channels which can be used, and consider the matched filter output of one channel at the central receiver. The performance of this channel (in terms of bit error rate) will depend on the probability distribution of the matched filter output. This output will consist of a desired component, due to the signal on the desired channel and undesired components caused by noise and interfering channels.

For the moment ignore interfering packets on the desired channel. The matched filter output voltage, sampled at the bit synchronous moment, can be represented as

$$Z = \sqrt{E_d} + n + V_I$$

where

E_d = energy received in desired channel per bit

n = noise sample

V_I = interference voltage sample.

The interference is made up of the sum of several signals of random phase and amplitude, on the competing channels. To the extent that the channels are independent, and that there are many of them, we can invoke the central limit theorem to assume that V_I is Gaussian. If the noise is also Gaussian, then Z will be a Gaussian random variable, and the bit error rate of bit decisions based on Z will be determined by the generalized signal-to-noise ratio d^2:

$$d^2 = \frac{E(Z)^2}{var(Z)}$$

Substituting for mean and variance we have

$$d^2 = \frac{E_d}{N_o + \overline{V_I^2}} \quad .$$

Because of the lack of synchronization of the channels, the interfering energy caused by a single channel will vary inversely with N. There will be N_c of the interfering waveforms, so that

$$\overline{V_I^2} = \frac{N_c}{N} \overline{E_I}$$

where $\overline{E_I}$ = average energy per bit of interfering channel considering duty-cycle and range variations.

To evaluate $\overline{E}_I$, assume that each terminal radiates energy received at the central receiver which is given by $\frac{E_t}{r^2}$ where r = range to terminal. Notice that we have assumed that propagation losses vary as $\frac{1}{r^2}$. We will consider a more general case later. Suppose that the terminals are uniformly distributed in a ring about the repeater with minimum range ϵ and maximum range R_o. Notice that ϵ cannot be zero. Then the average power, when the channel is on, is

$$E\left\{E_I \mid ON\right\} = \int_{\epsilon}^{R_o} \frac{E_t}{r^2}\, p(r)\, dr$$

where

$$p(r) = \frac{2r}{R_o^2 - \epsilon^2} \qquad \epsilon \le r \le R_o$$

$$= 0 \qquad \text{else} \quad .$$

so that

$$E\left\{E_I \mid ON\right\} = \frac{E_t}{R_o^2\left(1 - \frac{\epsilon^2}{R_o^2}\right)} \log\left(\frac{R_o^2}{\epsilon^2}\right)$$

If G = individual channel traffic then the duty-cycle is $1 - e^{-G}$ so that

$$\overline{E}_I = E\left\{E_I \mid ON\right\}(1 - e^{-G})$$

Thus for a terminal at R_o

$$d^2 = \frac{E_t/R_o^4}{N_o + \frac{N_c}{N}\frac{}{R_o^2\left(1 - \frac{\epsilon^2}{R_o^2}\right)}} \log\left(\frac{R_o^2}{\epsilon^2}\right)(1 - e^{-G})$$

For acceptable bit error rate, the system must be designed so that $d^2 > d_{min}^2$ where d_{min}^2 depends on the modulation type, but $d_{min}^2 \cong 10$.

Suppose that the system is designed so that the d^2 at R_o is equal to some multiple (α) of d^2_{min} in the absence of interference (this is an inefficient design, but necessary for multichannel operation).

Then

$$\frac{E_t}{N_o R_o^2} = \alpha\, d^2_{min}$$

and

$$N_c \leq \left(\frac{\alpha - 1}{\alpha}\right) \frac{N}{d^2_{min}} \frac{\left(1 - \frac{\epsilon^2}{R_o^2}\right)}{\log\left(\frac{R_o^2}{\epsilon^2}\right)} \left(\frac{1}{1 - e^{-G}}\right)$$

In this analysis we have used an inverse square-law propagation loss. In fact, measurements at 400 MHz and above show that the propagation loss varies as $\frac{1}{r^\beta}$ where β is usually greater than 3 and often approaches 4 for rough terrain or in urban areas. If we solve for N_c using an arbitrary $\beta > 2$, we find

$$N_c \leq \left(\frac{\alpha - 1}{\alpha}\right) \frac{N}{d^2_{min}} \frac{\beta - 2}{2} \left(\frac{\epsilon}{R_o}\right)^2 \frac{\left[\left(\frac{R_o}{\epsilon^2}\right)^2 - 1\right]}{\left[\left(\frac{R_o}{\epsilon}\right)^{\beta-2} - 1\right]} \left(\frac{1}{1 - e^{-G}}\right)$$

For $\frac{R_o}{\epsilon}$ greater than about 2, we find that inverse square-law propagation loss allows more channels than higher values of β, so if we use the inverse square-law we present results which are optimistic in favor of spread-spectrum. Inverse fourth-power loss would present pessimistic results.

Notice that in any case the number of channels which can operate simultaneously in the same band depends on two parameters which measure the inefficiency of power utilization. The first is α, which is the ratio of terminal energy-per-bit needed when operating multichannel to energy-per-bit needed when operating single-channel. This parameter must be greater than unity for multichannel operation, thus multichannel

operation requires reduction in bit rate for fixed power terminals. (This result has been shown only for code-division multichannel operation).

The second parameter is $\frac{\epsilon}{R_o}$, the ratio of minimum repeater-terminal range to maximum repeater-terminal range. The minimum range must be a significant fraction of the maximum range for multichannel operation. In some geometries this is no limitation, as for example when the repeater is on a mountain at some distance from the area being served. However, when the repeater is surrounded by terminals, ϵ will be necessarily small, so that either R_o will also be small, or $\frac{\epsilon}{R_o}$ will be small, and multi-channel operation will be impossible.

V COMPARISON OF SPREAD SPECTRUM AND NARROW BAND SYSTEMS

A. Criteria

There are many tradeoff dimensions to be considered when comparing alternative packet radio modulation formats. We have briefly touched on four of these:

(1) Terminal radiated power

(2) RF bandwidth used

(3) Throughput data rate

(4) Delay.

To these we should add at least two more.

(5) Compatibility with existing systems

(6) Vulnerability to enemy actions.

Both, or either of these last two may be so critical that we may ignore the first four; however, both are difficult to define until an application has been chosen. Leaving these last two criteria to a later time, we are left with devising some means to present the relationship of the first four tradeoff dimensions.

We have already presented curves of throughput and delay for several single-channel schemes using narrow band or spread-spectrum signals (ALOHA and slotted ALOHA, etc.). In order to draw comparable curves for multi-channel schemes we must take into account the r.f. bandwidth used as well as the area covered by a fixed transmitter power. It seems reasonable to fix the transmitter power at some arbitrary level and then determine the area that a narrow-band system could cover. If we then adjust the spread-spectrum system so that it covers the same area we will have comparable systems. Once we have comparable systems, we can compare curves of normalized delay versus normalized throughput, where delay is measured in narrowband packet durations and throughput is measured in information bits per unit bandwidth. To assure that we compare systems with the same total capacity we will adjust the narrow band system delay versus

throughput relations to account for the inefficiency encountered due to the necessity to provide frequency guard-bands when cascading narrow band channels.

B. Coverage Areas and Bit Rates

In order to use multiple spread-spectrum channels we have found that we must transmit more energy per bit than an equivalent-performance narrow band channel. For a fixed transmitter power, this means that either the bit duration must be extended, or the maximum range reduced. Since we wish to compare systems with equal coverage areas we must reduce the data rate of the spread-spectrum system. The amount of reduction is determined as follows.

The area covered by the spread-spectrum system is

$$A_s = \pi R_o^2 - \pi \epsilon^2$$

$$= \pi R_o^2 \left(1 - \frac{\epsilon^2}{R_o^2}\right)$$

The area covered by the narrow band system is

$$A_n = \pi R^2 \quad .$$

The energy per bit-to-noise density ratios are

$$d_s^2 = \frac{kPT_{ss}}{R_o^2 N_o}$$

and

$$d_n^2 = \frac{kPT_n}{R^2 N_o}$$

where

P = peak transmitted power

N_o = noise power density

k = constant to account for antenna gains and receiver noise figures

T_n = bit duration of narrowband system

Tss = bit duration of spread-spectrum system.

We require that

$$d_s^2 = \alpha d_n^2$$

and

$$A_s = A_n$$

so that

$$T_s = \frac{\left(1 - \frac{\epsilon^2}{R_o^2}\right)}{\alpha} T_n \quad .$$

C. Delay and Throughput

To compare normalized throughput we normalize by the total bandwidth occupied. If S is the throughput of a spread-spectrum mode, and S_s is the normalized multi-channel throughput, then for N_c channels in a band which is N times as wide as that needed to accommodate a single channel with bit duration T_{ss}, we have

$$S_s = \frac{N_c S}{N} \quad .$$

From our previous results for N_c we find that for small G S_s is given by

$$S_s = \frac{S}{G} \quad \frac{1}{d_{min}^2} \quad \frac{\alpha - 1}{\alpha} \quad \frac{1 - \frac{\epsilon^2}{R_o^2}}{\log (R_o^2/\epsilon^2)}$$

But we know that G/S is the average packet delay (in units of packet-time at the bit rate $\frac{1}{T_s}$. We can find the delay D_s in delay units of packet-time at the bit rate $\frac{1}{T_n}$ from our previous relation for T_{ss} and T_n.

$$D_s = \frac{G}{S} \frac{\alpha}{\left(1 - \frac{\epsilon^2}{R_o^2}\right)}$$

Substituting this in the equation for S_s results in

$$S_s = \left(\frac{1}{D_s}\right)\left(\frac{1}{d_{min}^2}\right)\left(\frac{\alpha - 1}{\log (R_o^2/\epsilon^2)}\right)$$

Notice that throughput is maximized when delay is minimized. The minimum delay that can be achieved will be determined by the choice of ϵ/R_o and α as well as the access mode used. In no case can S ever exceed G, hence a lower bound on D_s is

$$D_s \geq \frac{\alpha}{1 - \frac{\epsilon^2}{R_o^2}}$$

so that if we assume operation at this point we will have an optimistic bound on the delay versus throughput achievable with any multiple channel spread-spectrum mode. Since this bound has been derived using the fact that multiple channels will interfere with each other unless they are synchronized (an impossible task), we call this the interference bound. It is plotted for several values of α and ϵ/R_o in Figure 7.

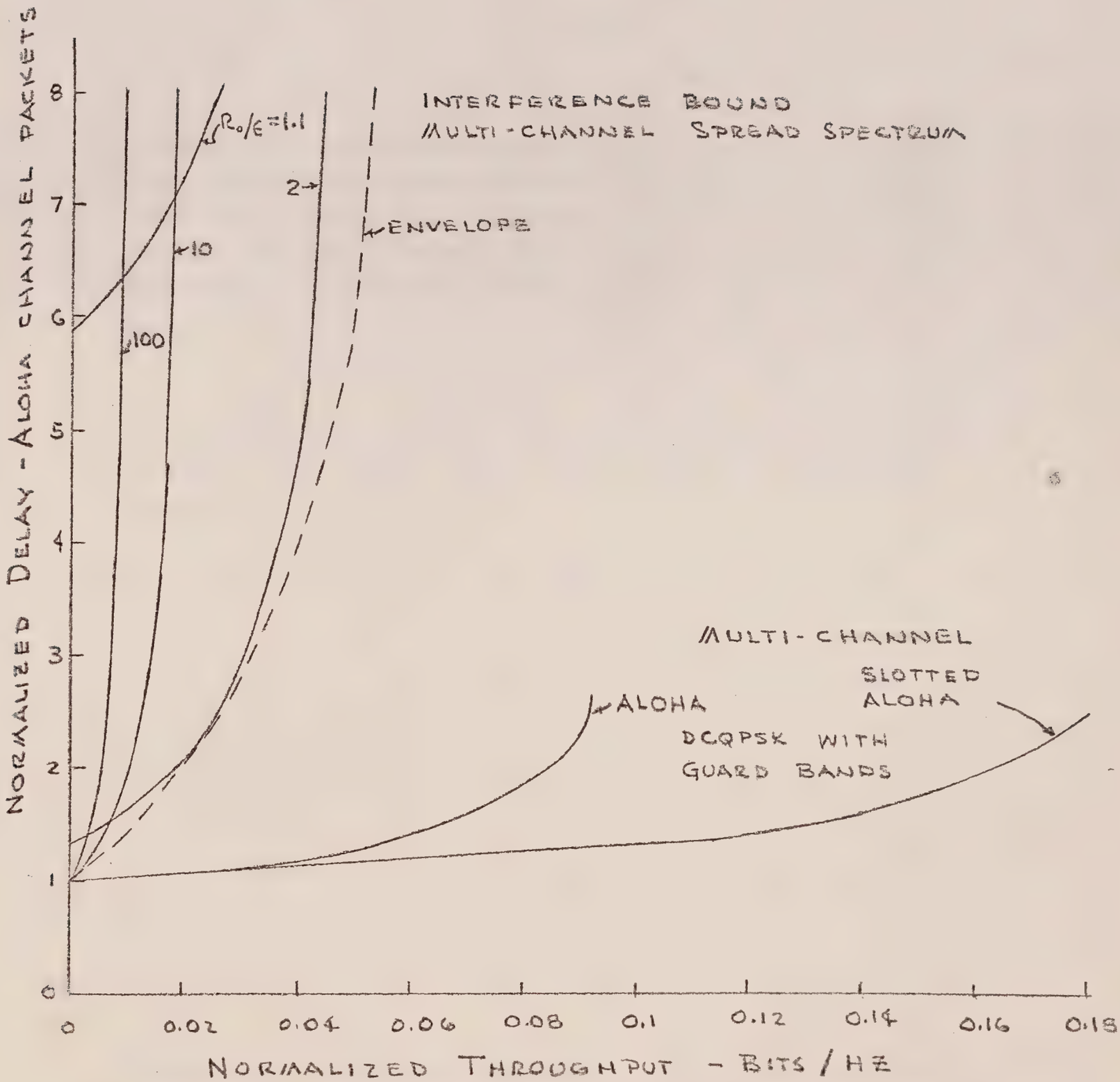


Figure 7 DELAY THROUGHPUT TRADEOFFS FOR MULTI-CHANNEL SYSTEMS

For comparison purposes we have also plotted the throughput-delay curves for multiple narrow-band ALOHA and slotted ALOHA channels. For ALOHA we recall that

$$S = Ge^{-2G}$$

so that $D = \frac{G}{S} = e^{-2G}$. But if we stack several such channels we require frequency guard-bands between them to assure no interference. A pessimistic bound for such guard bands would double the bandwidth of the channel. Most likely we could space the channels much closer together, but this bound is certainly achievable. The multi-channel bandwidth-normalized throughput will be related to single channel throughput by

$$S_n = \frac{S}{2}$$

since each channel will require twice the bandwidth of a single channel. The multi-channel delay, however, will not change, so that the two equations

$$S_n = \frac{Ge^{-2G}}{2}$$

$$D_n = e^{-2G}$$

describe the throughput-delay characteristics of a set of ALOHA channels stacked in frequency.

Similarly

$$S_n = \frac{Ge^{-G}}{2}$$

and

$$D_n = e^{-G}$$

describe a set of slotted ALOHA channels. Both of these curves are plotted in Figure 7.

REFERENCES

(1) N. Abramson, "The ALOHA System--Another Alternative for Computer Communications, "Fall Joint Computer Conference (1970).

(2) G. L. Turin et al., "A Statistical Model of Urban Multipath Propagation," IEEE Trans. on Veh. Tech., pp. 1-9 (February 1972).

(3) D. C. Cox, "Time-and Frequency-Domain Chracterizations of Multipath Propagation at 910 MHz in a Suburban Mobile-Radio Environment," Radio Science, Vol. 7, No. 12, pp. 1069-1077 (December 1972).